# Recent progress on the QCD phase diagram and the equation of state

Gergely Endrődi

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# Recent progress on the QCD phase diagram and the equation of state

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in collaboration with:

Bielefeld: Bastian Brandt, Eduardo Garnacho, Javier Hernández, Gergely Markó, Laurin Pannullo, Leon Sandbote, Dean Valois

Frankfurt, Darmstadt: crc-tr211.org

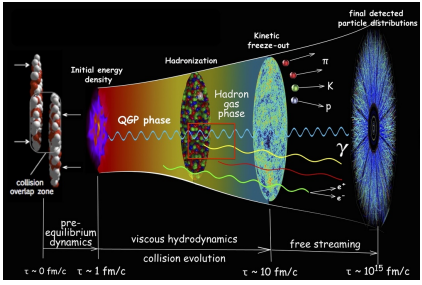
# Outline

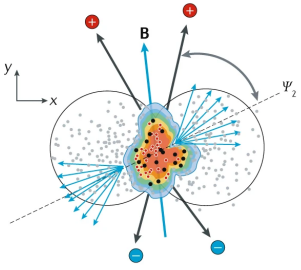
- introduction: strongly interacting matter in the presence of
  - strong electromagnetic fields
  - nonzero isospin-asymmetry
- lattice simulation techniques
- phase diagrams: current status
- further electromagnetic effects
  - magnetized and dense systems
  - topology and chirality
  - inhomogeneities
- summary

# Introduction

# Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^{\circ}C = 200$ MeV, $n \lesssim 0.12$ fm$^{-3}$
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$



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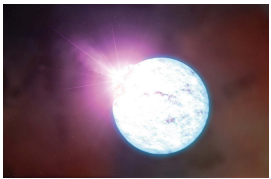

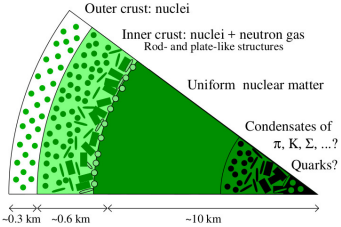


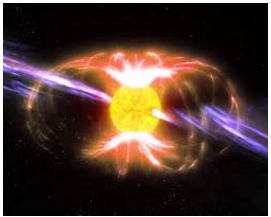

Lattimer, Nature Astronomy 2019

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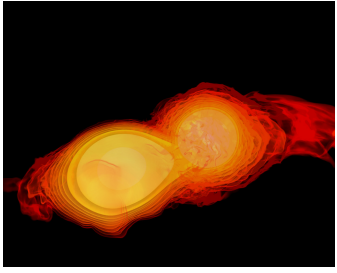

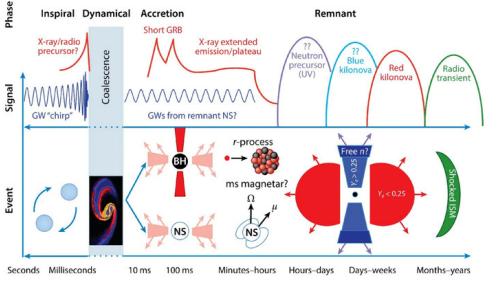

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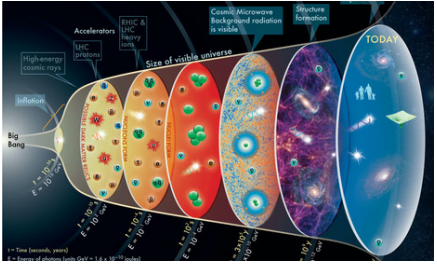
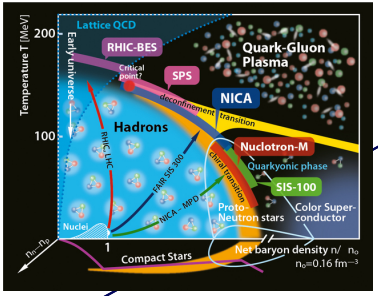
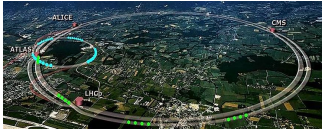
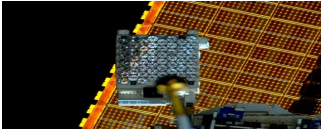
- neutron star mergers $T \lesssim 50$ MeV
- eary universe, QCD epoch $T \lesssim 200$ MeV
  standard scenario: $n \approx 0$ also allowed: $n_Q = 0$, $n_\ell/s \lesssim 0.01$



# Major experimental and observational campaigns

Observational astronomy

Heavy ion collisions

image courtesy F. Cuteri

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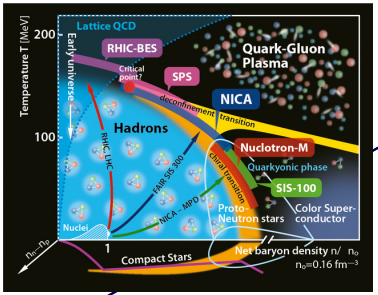


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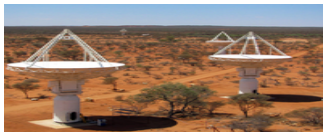

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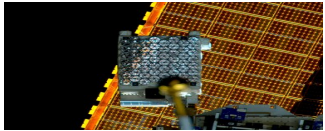

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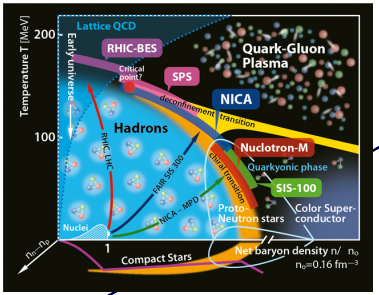


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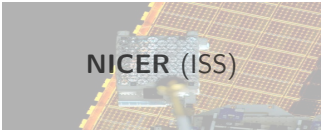


image courtesy F. Cuteri

# QCD phase diagram(s)

# Phase diagram

- control parameters: $T$, $n \leftrightarrow \mu$, $B$ $\qquad \mu_{\{u,d,s\}}$ / $\mu_{\{B,Q,S\}}$ / $\mu_{\{B,I,S\}}$

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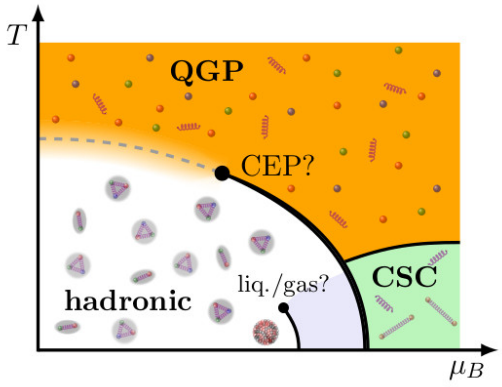

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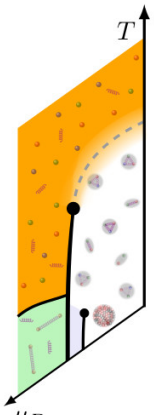

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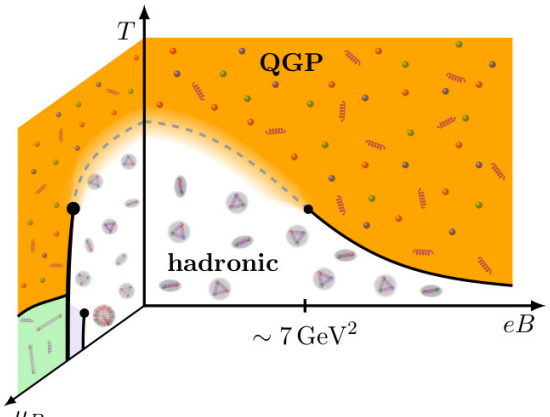

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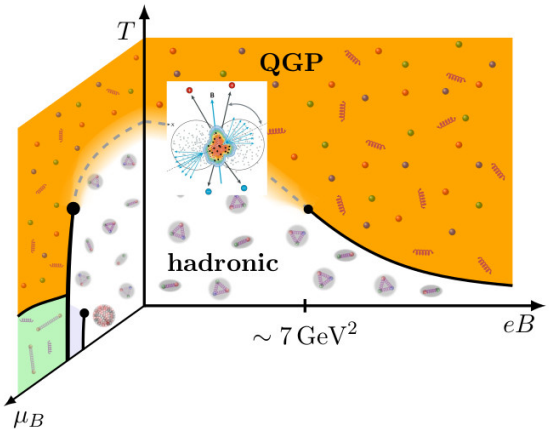

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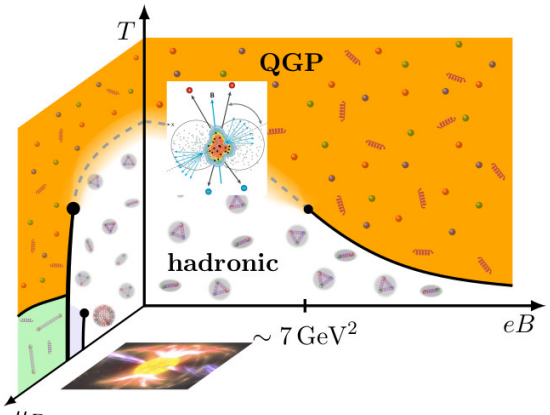

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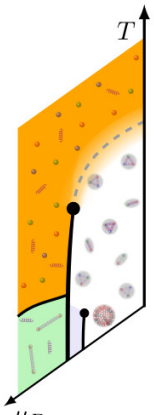

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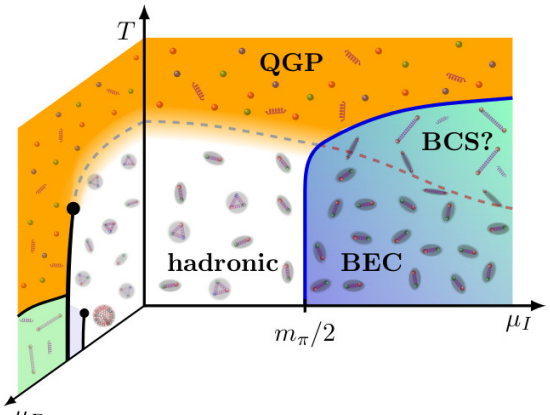

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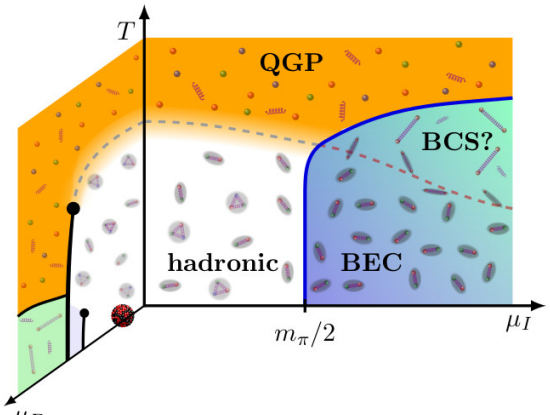

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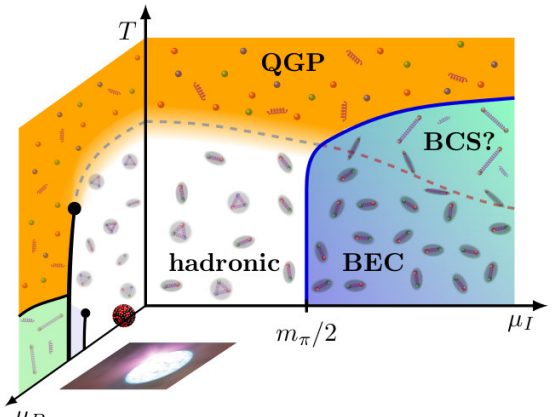

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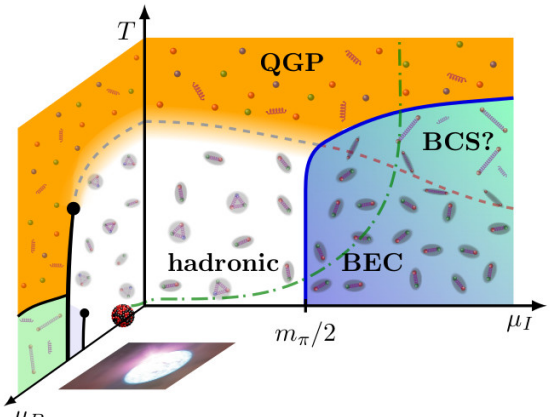


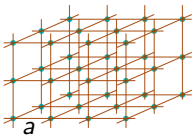
# Lattice QCD simulations

# Lattice simulations

- path integral — Feynman '48

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left( - \int \mathrm{d}^4 x \, \mathcal{L}_{\mathrm{QCD}}(x) \right)$$

- discretize QCD action on space-time lattice — Wilson '74

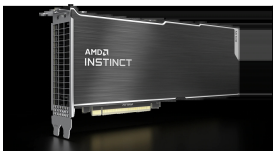



continuum limit $a \to 0$ in a fixed physical volume: $N \to \infty$

- dimensionality of lattice path integral: $10^{9-10}$ $\rightsquigarrow$ need for parallel computing

SuperMUC-NG

nvidia.com




amd.com

Bielefeld GPU cluster

# Monte Carlo simulations

- Euclidean QCD path integral over gauge field $\mathcal{A}$

$$\mathcal{Z} = \int \mathcal{D}\mathcal{A}\, e^{-S_G[\mathcal{A}]}\, \det[\not{D} + m]$$

- Monte-Carlo simulations need: $\det[\not{D} + m] \in \mathbb{R}^+$
  for that one needs $\Gamma$ so that

$$\Gamma \not{D} \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

$$\det[\not{D} + m] = \det[\Gamma^\dagger \Gamma(\not{D} + m)] = \det[\Gamma(\not{D} + m)\Gamma^\dagger] = \det[\not{D}^\dagger + m] = \det[\not{D} + m]^*$$

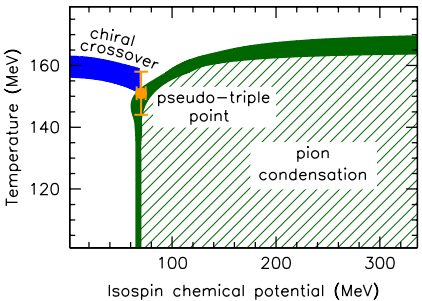
- usually positivity can also be shown
- such a $\Gamma$ exists: $B$, $\mu_I$, $i\mu_B$ ✓
- no $\Gamma$ exists: complex action problem $\mu_B$ ↯

# Isospin-asymmetry

# Phase diagram

- phases in the $T - \mu_I$ phase diagram: hadronic (confined), quark-gluon plasma (deconfined), pion condensation (confined)

  Brandt, Endrődi, Schmalzbauer '17  Brandt, Endrődi '19



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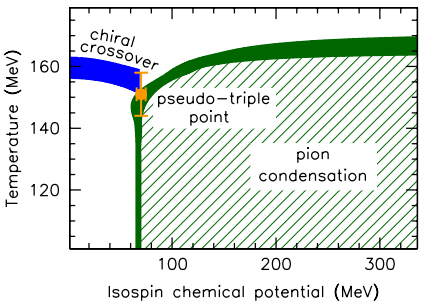


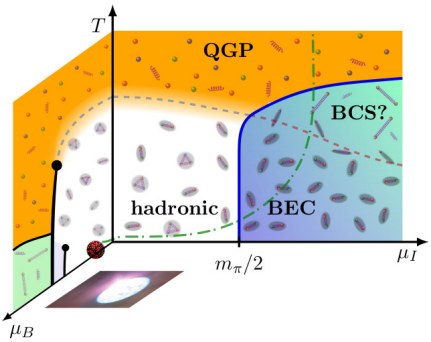

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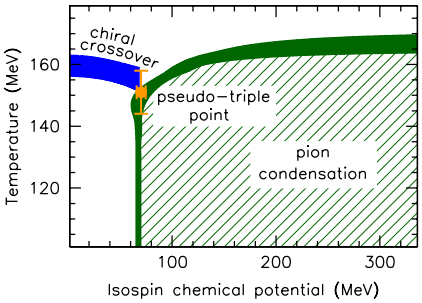
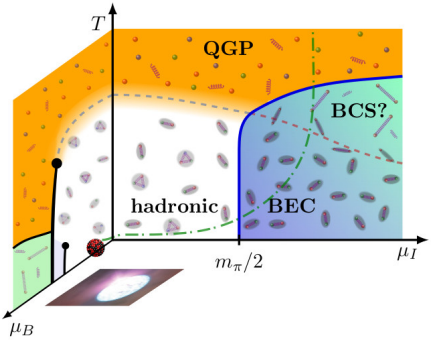



- comparison to effective models, $\chi$PT, Q2CD, ...

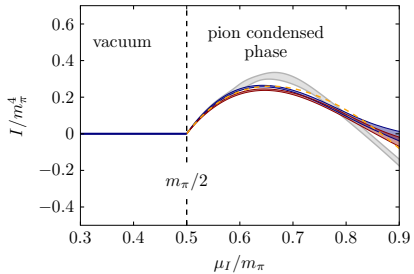
  Adhikari et al. '18  Zhokhov et al. '19  Adhikari et al. '20  Boz et al. '20

  Astrakhantsev et al. '20  Andersen et al. '23  von Smekal et al. '19

# Equation of state

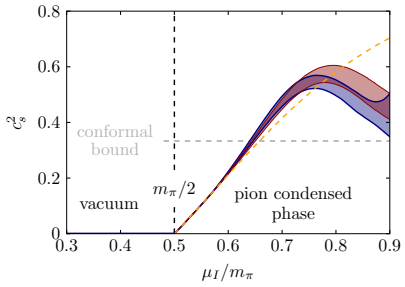
- interaction measure $I = \epsilon - 3p$ negative for low $T$ and high $\mu_I$

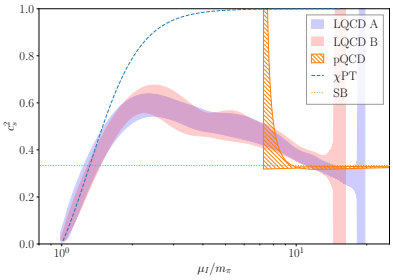
  Brandt, Cuteri, Endrődi '22



# Equation of state

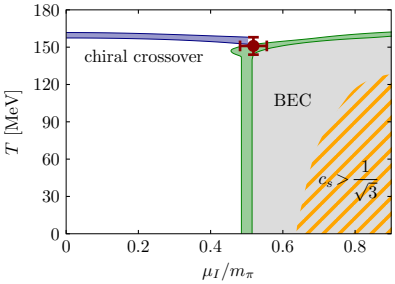
- interaction measure $I = \epsilon - 3p$ negative for low $T$ and high $\mu_I$
  - Brandt, Cuteri, Endrődi '22
- speed of sound $c_s^2 = \frac{\partial p}{\partial \epsilon}\Big|_{s/n_I}$ above conformal limit 1/3 for low $T$ and high $\mu_I$
  - Brandt, Cuteri, Endrődi '22  Abbott et al. '23



# Equation of state

- EoS gets very stiff inside pion condensation phase
- 'supersonic' region of pion condensate



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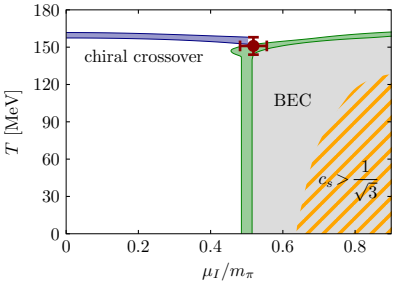


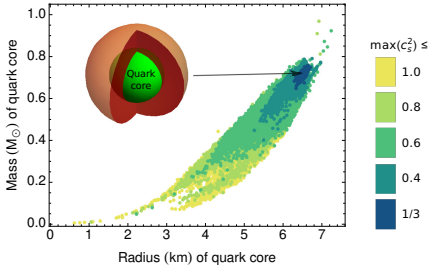

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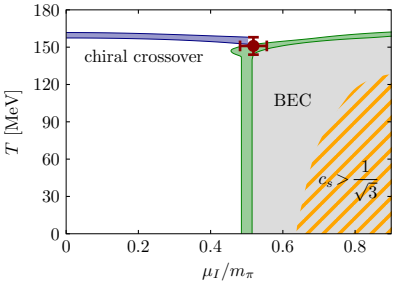
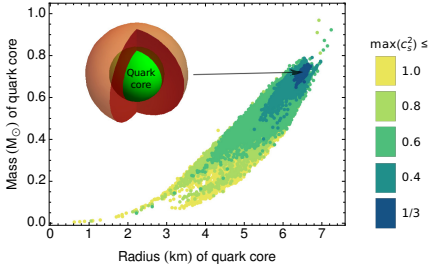
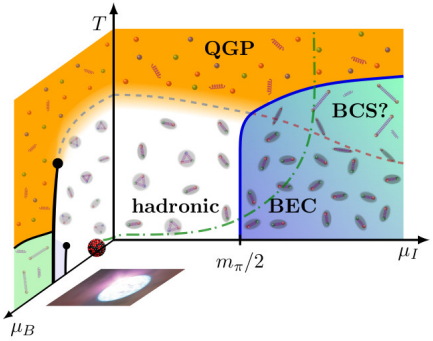



- comparison: $\chi$PT Adhikari et al. '21 models Avancini et al. '19 FRG Braun, Schallmo '22 $\chi$FT Leonhardt et al. '20

# Combining different conserved charges

- explore three-dimensional phase diagram



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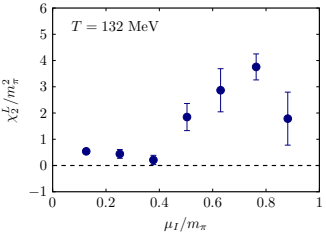

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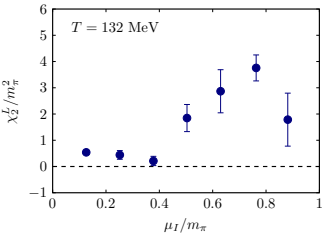


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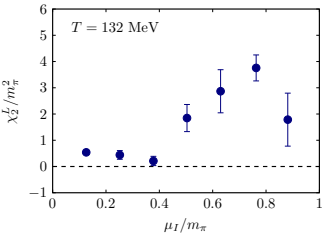


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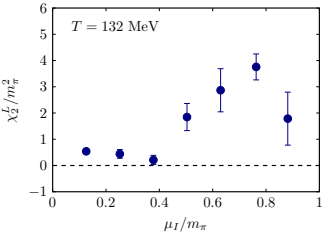


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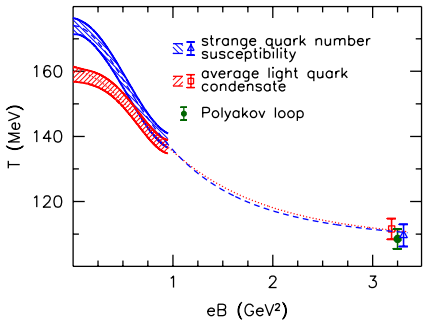
- impact of isospin-asymmetry on dense quarkyonic matter Max Moss poster

# Magnetic fields

# Magnetic phase diagram

- QCD crossover temperature in the phase diagram

  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12 Endrődi '15



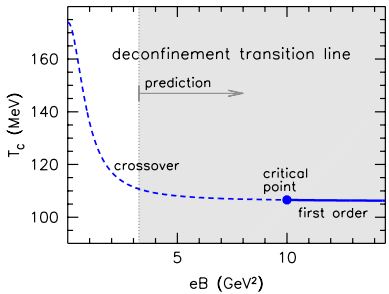

- $T_c$ is reduced by $B$

  contrary to almost all effective theories and low-energy models of QCD

  Andersen, Naylor, Tranberg '14

# Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order transition $\Rightarrow$ critical point!
  Miransky, Shovkovy '02
- estimate based on intermediate fields $\Rightarrow$ $eB_c \approx 10(2)$ GeV$^2$ Endrődi '15



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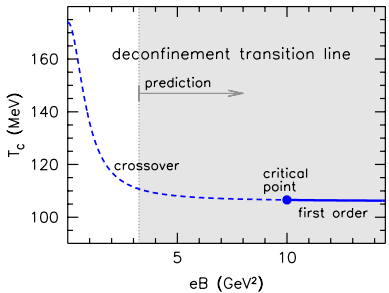


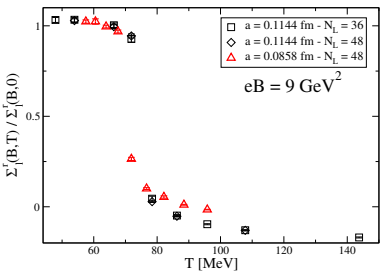

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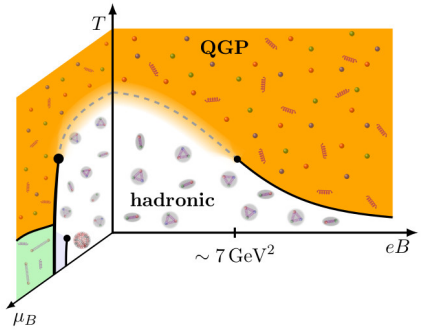
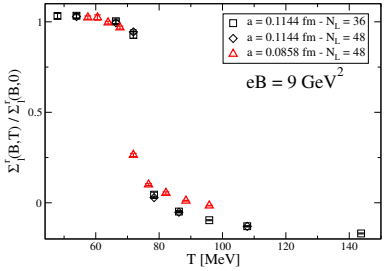
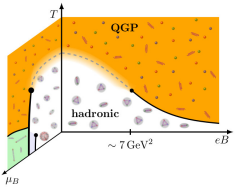



- first ever lattice evidence for first-order phase transition in QCD at physical masses and physical parameters!

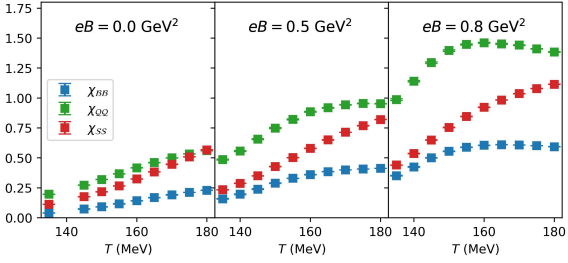
# Magnetic fields and nonzero density




- relationship between the critical points?

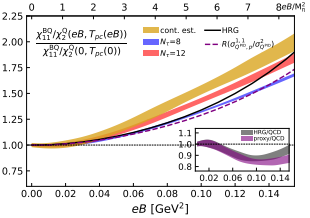
- recent lattice simulations with $B > 0$ and $\mu_B > 0$
  via $i\mu_B$ simulations Braguta et al. '19 Astrakhantsev et al. '24 Valois et al. '23
  via Taylor-expansion Ding et al. '21 Ding et al. '23

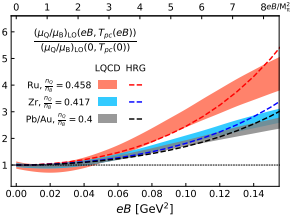
- ensuring the conditions $n_S = 0$, $n_Q/n_B = 0.4$ more challenging Valois et al. '23



# Magnetic fields in heavy-ion collisions

- how to measure $B$ in heavy-ion collisions?
- suggestions for observables to use as magnetometers — Ding et al. '23



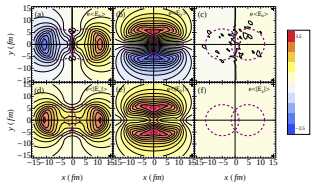




- Hadron Resonance Gas model at nonzero magnetic fields — Endrődi '13
- issues with HRG to match to experimental yields
  Marczenko et al. '24 Vovchenko et al. '24 Vovchenko Wed 11:40 Bulk&Phase
- estimating the magnetic field in HIC
  via virtual photon polarization and dilepton anisotropy Minghua Wei poster
  via heavy quark spin polarization Sharma Wed 9:50 Chen Wed 11:20 Bulk&Phase

# Beyond constant magnetic fields: inhomogeneities

# Inhomogeneous magnetic fields

- off-central heavy-ion collisions: inhomogeneous magnetic fields Deng et al. '12



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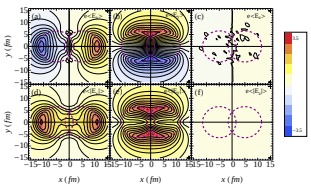

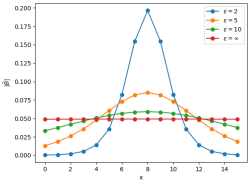


- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04

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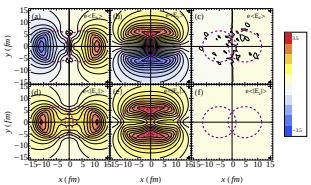
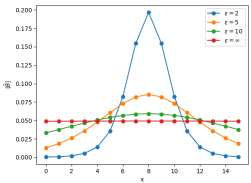
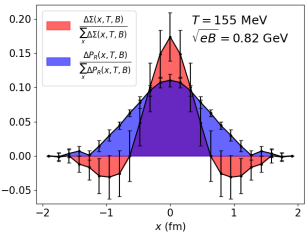



- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ — Dunne '04
- impact: condensate, Polyakov loop

  Brandt, Cuteri, Endrődi, Markó, Sandbote, Valois '23



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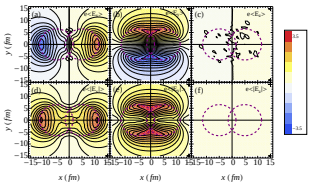
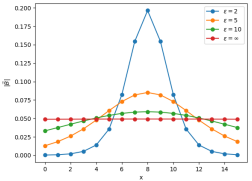
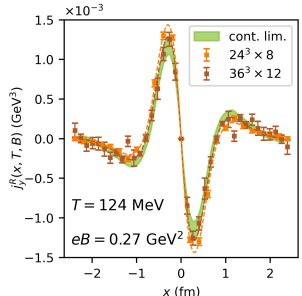





- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ — Dunne '04
- impact: electric current

  — Brandt, Endrődi, Markó, Valois '24



# Magnetic fields and chiral imbalance: anomalous transport

# Anomalous transport

- chiral magnetic effect (CME): Fukushima, Kharzeev, Warringa '08
  vector current due to magnetic field and (low) chirality

$$\langle \vec{J} \rangle = C_{\mathrm{CME}} \cdot \mu_5 \cdot \vec{B}$$

- chiral separation effect (CSE): Son, Zhitnitsky '04
  axial current due magnetic field and (low) density

$$\langle \vec{J_5} \rangle = C_{\mathrm{CSE}} \cdot \mu \cdot \vec{B}$$

- experimental searches for CME and related observables STAR collaboration '21
- difficult to remove noise and $v_2$-related effects Han-Sheng Li poster

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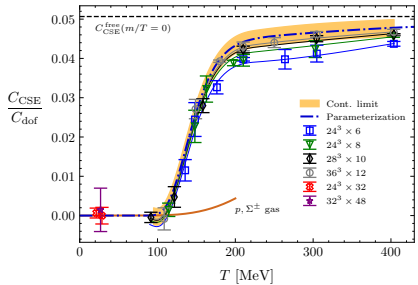
- must distinguish **in-equilibrium** and **out-of-equilibrium** effects

# CSE from lattice QCD

- first determination of **in-equilibrium** CSE coefficient with continuum extrapolated lattice simulations Brandt, Endrődi, Garnacho, Markó, '23




- $C_{\text{CSE}} = 1/(2\pi^2)$ for high $T$
- CSE is suppressed in hadronic phase (see also Buividovich, Smith, von Smekal '21 )
- $C_{\text{CSE}}(T)$ is a good measure for chiral symmetry restoration

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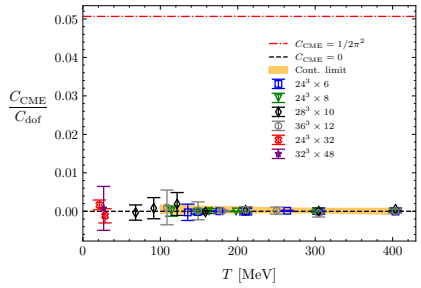

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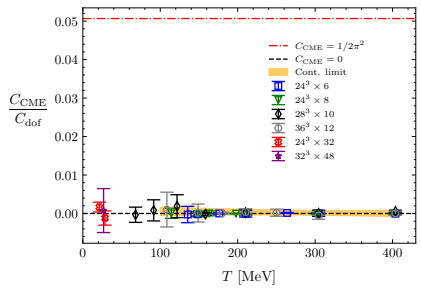


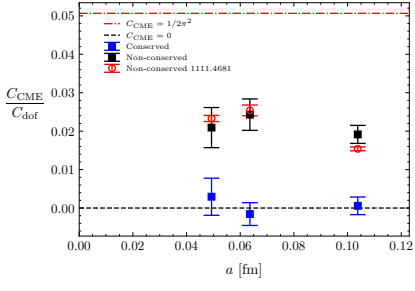


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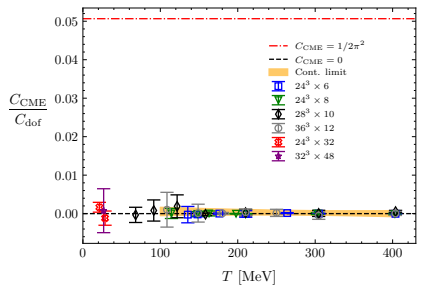


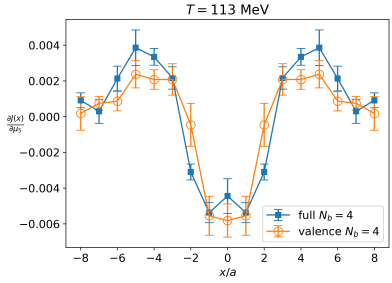


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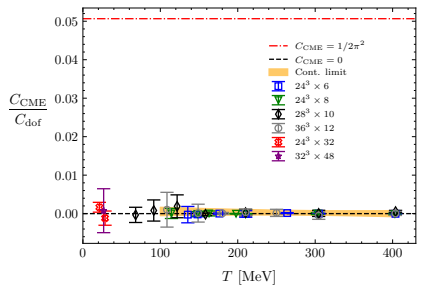


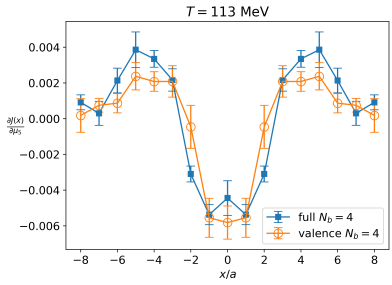


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- non-trivial response in inhomogeneous field $B(x)$
- this is not the **out-of-equilibrium** effect

# Summary

# Summary

- $T - \mu_I$ phase diagram and pion condensation
- faster-than-conformal sounds
- $T - B$ phase diagram and the critical point
- in-equilibrium anomalous transport phenomena from lattice QCD