Abstract: We introduce graphs associated to transport problems between discrete marginals, that allow to characterize the set of all optimizers given one primal optimizer. In particular, we establish that connectivity of those graphs is a necessary and sufficient condition for uniqueness of the dual optimizers. Moreover, we provide an algorithm that can efficiently compute the dual optimizer that is the limit, as the regularization parameter goes to zero, of the dual entropic optimizers. Our results find an application in a Stackelberg-Cournot-Nash game, for which we obtain existence and characterization of the equilibria.