

# Evolutionary Economics and Chaos Theory

## New Directions in Technology Studies

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**P I N T E R**  
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## 4 A Note on Competition among Techniques in the Presence of Increasing Returns to Scale

*Alfred Greiner and Friedrich Kugler*

Recently Amable (1992) presented an evolutionary model in which he investigates the diffusion process of two competing technologies producing the same good for a certain market. As a result he finds out that the dynamic system describing the evolution of the produced goods always converges to a rest point where the market shares of product 1 and 2 stay constant over time. The respective amounts of the goods produced depend on the rate of returns to scale prevailing in the production process. In this paper we show that this result changes drastically if we use a discrete time framework instead of a continuous one as does Amable. Then convergence to a rest point need no longer necessarily hold and regular or even erratic fluctuations of the amounts produced may occur.

The outcome of completely different results is due to the fact that we create a completely different economic model by employing a discrete-time formulation. Therefore, it is clear that the substitution of a first-order difference equation by a first-order differential equation does not yield the same dynamic outcome — Sparrow (1980) and Medio and Gallo (1989) have shown how to transform a system with a fixed delay into a continuous one such that the dynamic behaviour is preserved. It is a questionable assumption that economic agents would not take their decisions continuously. But on the other hand, there are many models in the economic literature which use a discrete-time formulation (see, for example, volume 40 of the *Journal of Economic Theory* of 1986). There are even economists asserting that a discrete-time framework is preferable since decisions of economic agents only occur at certain points in time — cf. the book review by Eckalbar (1992) who mentions six arguments in favour of discrete-time modeling. For these reasons we assert that the model presented by Amable does also make sense in discrete time and therefore investigate the dynamic properties of this new model. For a further discussion of the nature of time see, for example, Gandolfo (1981).

Let us now come to the economic model. Amable considers a market for two goods  $(y^1, y^2)$ , which are perfect substitutes for each other. The evolution of the

demand for these goods in discrete time is given by the logistic pattern

$$y_{t+1}^1 = y_t^1 + \beta y_t^1 (D - y_t^1 - y_t^2) \quad (4.1)$$

$$y_{t+1}^2 = y_t^2 + \beta y_t^2 (D - y_t^1 - y_t^2) \quad (4.2)$$

where  $\beta$  is a positive imitation coefficient and  $D$  is the maximum attainable level of demand for the goods. The level of saturation  $D$  is determined endogenously:

$$D = a_0 - a_1 p \quad a_0 > 0; a_1 > 0 \quad (4.3)$$

where  $p$  is the price which is the same for the two goods.

The growth rate of production capacity for the technique used to produce good  $i$ ,  $x_i^j$ , is assumed to be a positive function of unit profit  $\Pi_i^j$ :

$$\frac{x_{t+1}^i - x_t^i}{x_t^i} = \sigma \Pi_i^j \quad \sigma > 0 \quad (4.4)$$

$\Pi_i^j$  is defined as

$$\Pi_i^j = p - c^i = p - c_0^i - c_1^i x_i^j \quad c_0^i > 0, c_1^i \neq 0. \quad (4.5)$$

Assuming that demand equals supply, the equilibrium condition is  $x_i^j = y_i^j$  for all  $t$ .

By combining equations (4.4) and (4.5), solving for  $p$  and substituting the result in equation (4.3) and the  $D$  in equation (4.1) and (4.2), the evolution of  $x_i^j$  and  $x_{t+1}^j$  is given by

$$x_{t+1}^1 = x_t^1 + A_1 x_t^1 (B_1 - x_t^1 - C_1 x_t^2) \quad (4.6)$$

$$x_{t+1}^2 = x_t^2 + A_2 x_t^2 (B_2 - x_t^2 - C_2 x_t^1) \quad (4.7)$$

with

$$A_i = \frac{\sigma \beta (1 + a_1 c_1^i)}{\sigma + a_1 \beta}, B_i = \frac{a_0 - a_1 c_0^i}{1 + a_1 c_1^i}, C_i = \frac{1}{1 + a_1 c_1^i} \quad i = 1, 2$$

In what follows we will confine our investigations to the case where the saturation levels for the two techniques,  $B_1, B_2$ , are positive, an assumption which makes sense economically. Moreover, we will allow for increasing returns to scale, that is,  $c_1^i < 0$ , but assume that they are not too strong such that  $C_i > 0$  holds.



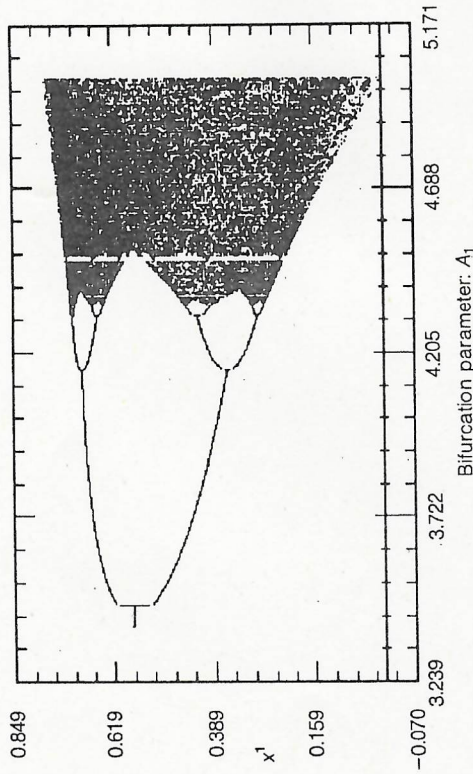


Figure 4.1 Bifurcation diagram for  $x_1^i$

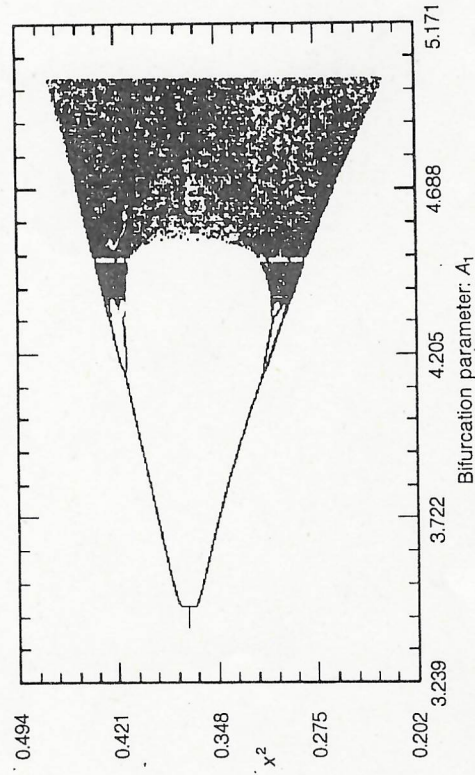


Figure 4.2 Bifurcation diagram for  $x_2^i$

As in the continuous-time case we can state that there are four equilibrium points in this model, that is, situations for which  $x_{t+1}^i = x_t^i$  holds for all  $t$  ( $i = 1, 2$ ), namely  $(0, 0)$ ,  $(0, B_2)$ ,  $(B_1, 0)$  and  $((B_1 - C_1 B_2)/(1 - C_1 C_2), (B_2 - C_2 B_1)/(1 - C_1 C_2))$ . Computing the eigenvalues of the Jacobian of the linearized system shows that the trivial solution  $(0, 0)$  is always unstable. Stability conditions for all other points may be determined technically but cannot be interpreted economically. Therefore, we will instantaneously resort to numerical simulation in order to demonstrate that any sort of dynamic behaviour will be possible in that model.

Using the equality  $A_1 C_1 = A_2 C_2$ , we can rewrite equations (4.6) and (4.7) as follows:

$$x_{t+1}^1 = x_t^1 + A_1 x_t^1 (B_1 - x_t^1 - C_1 x_t^2) \quad (4.8)$$

$$x_{t+1}^2 = x_t^2 + A_1 (C_1 / C_2) x_t^2 (B_2 - x_t^2 - C_2 x_t^1) \quad (4.9)$$

For the numerical example we make use of the following parameter values:  $C_1 = 0.06$ ;  $C_2 = 0.052$ ;  $B_1 = 0.6$ ;  $B_2 = 0.4$ .  $A_1$  serves as bifurcation parameter. The initial conditions are set to  $x_0^1 = 0.01$ ,  $x_0^2 = 0.02$ . Note that in this case both technique 1 and technique 2 show decreasing returns to scale. Computing the eigenvalues of the Jacobian of the linearized system then gives  $\lambda_1 = 1 + 0.567A_1$ ,  $\lambda_2 = 1 - 0.461538A_1$  for the steady state values  $(x^1, x^2) = (0, 0.4)$  and  $\lambda_1 = 1 + 0.425538A_1$ ,  $\lambda_2 = 1 - 0.6A_1$  for  $(x^1, x^2) = (0.6, 0)$ , showing that both equilibrium points are unstable (recall that  $A_1 > 0$ ). For the fourth equilibrium which is given by  $(x^1, x^2) = (0.5778027, 0.3699542)$  with the parameter values of our example we get  $\lambda_1 = 1 - 0.421933A_1$ ,  $\lambda_2 = 1 - 0.58274A_1$ ; that is, this point is stable if  $A_1 < 3.4320623$ .

Figures 4.1 and 4.2 show the bifurcation diagrams for  $x_1^i$  and  $x_2^i$ . It can be seen how the solution bifurcates for the value  $A_1 \approx 3.43$ .

Increasing the value of  $A_1$  further, we finally observe completely erratic fluctuations of the variables  $x_1^i$  and  $x_2^i$  for values of  $A_1$  greater than about 4.35. Now we have chaos, or more strictly speaking 'topological' chaos in the sense of Li and Yorke (1975).

In Figures 4.3-4.5 the time paths for  $x_1^i, x_2^i$  are presented for different values of  $A_1$  (the dotted line denotes  $x_2^i$ ). In Figure 4.3  $A_1 = 2.5$ , showing that the variables converge to their stationary values. In Figures 4.4 and 4.5  $A_1 = 4.25$  and  $A_1 = 5$ , revealing periodic oscillations of period 4 and completely aperiodic oscillations, respectively.

Figures 4.6-4.8 finally show strange attractors for different values of  $A_1$  (100 000 iterations). The values for  $A_1$  are  $A_1 = 4.53$ ,  $A_1 = 4.6$ ,  $A_1 = 5$  for Figures 4.6-4.8, respectively. The Lyapunov exponents corresponding to the



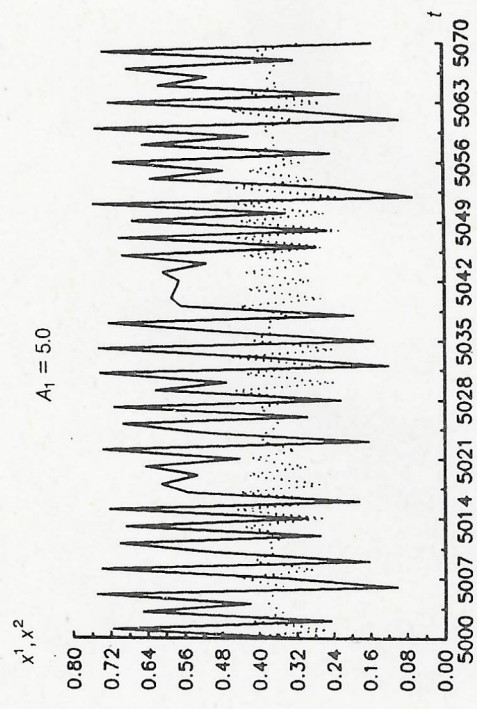


Figure 4.5 Erratic oscillations of  $x_1, x_2$

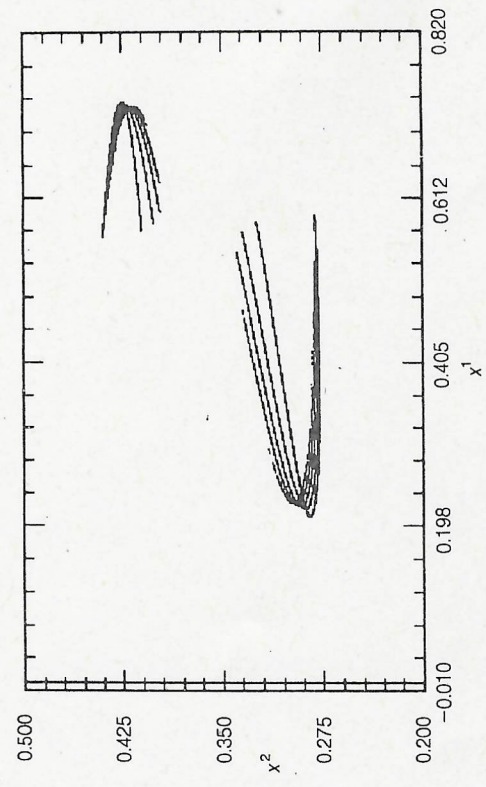


Figure 4.6 Strange attractor for  $A_1 = 4.53$

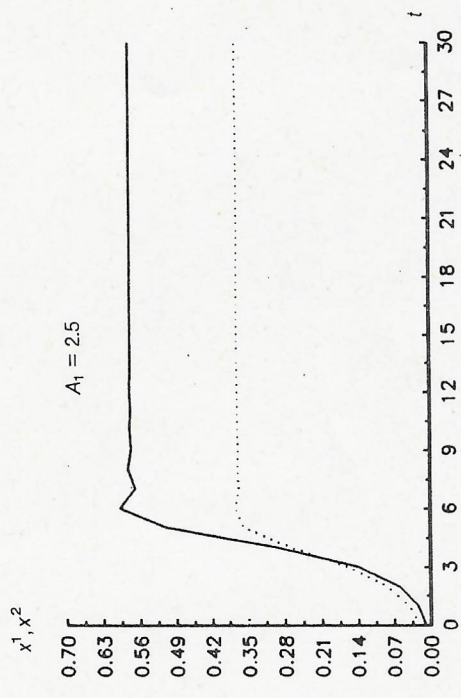


Figure 4.3 Constant time path for  $x_1, x_2$

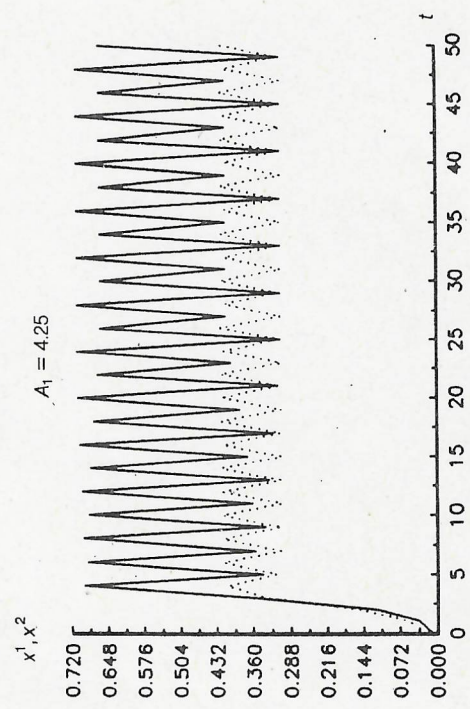
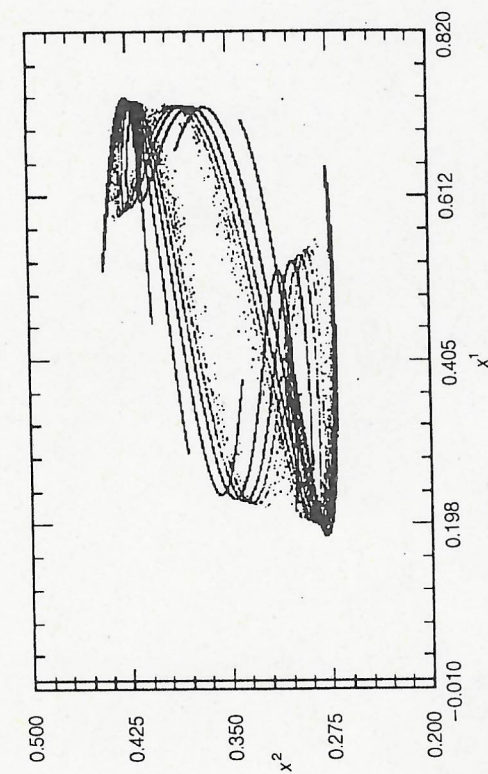
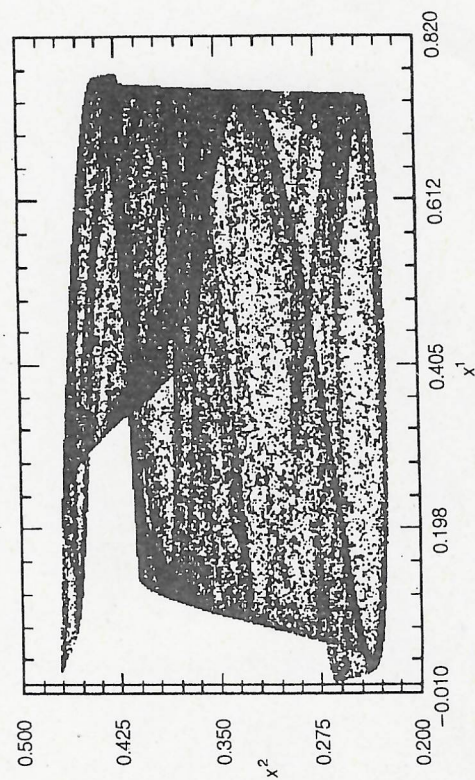


Figure 4.4 Cyclical oscillations of  $x_1, x_2$



Figure 4.7 Strange attractor for  $A_1 = 4.6$ Figure 4.8 Strange attractor for  $A_1 = 5.0$ 

values of  $A_1$  are 0.306 763,  $-1.0456$  for  $A_1 = 4.53$ ; 0.335 924,  $-0.695 448$  for  $A_1 = 4.6$ ; and 0.473 521,  $-0.188 07$  for  $A_1 = 5$ , confirming our results.

The economic interpretation of that result seems to be straightforward. Besides the possibility of an equilibrium solution with constant production and unchanging market shares, as in the original model, we observe that persistent fluctuations may be the outcome of that model where competition among the two techniques prevents the system from converging to a rest point. So, on the one hand, there may be regular, periodic cycles stating that production in that market occurs cyclically with the distribution of the market shares between technique 1 and technique 2 showing periodic fluctuations. In that case the period during which technique 1 will gain a larger share of the market than technique 2 can completely be predicted. On the other hand, there may be erratic fluctuations stating that production and the distribution of the market shares are subject to continuous and unpredictable change. In that case we may reason that none of the techniques is capable of achieving a position in which it will gain a higher share of the market than its competitor for a predetermined period of time.

We have seen that, all other parameters constant,  $A_1$  determines the stability properties of that model, with a more complex dynamic behaviour being obtained for larger values of that parameter. It can easily be shown that for a given cost structure ( $C_1$  constant)  $A_1$  positively depends on both the imitation coefficient  $\beta$  and on  $\sigma$ , the coefficient giving the dependence of the growth rate of the two production techniques on profit. Therefore we can state the following. The larger the influence of profit on the growth rate of production of technique 1 and technique 2 and the larger the imitation coefficient  $\beta$ , the more likely is the emergence of complex dynamics. The imitation coefficient  $\beta$  may also be termed a reaction coefficient, giving the speed of adjustment of demand for goods 1 and 2 to their saturation levels.

Our results were derived under the assumption of decreasing returns to scale in the two production technologies. Allowing for increasing returns, no concrete outcome could be observed. Simulation runs, however, suggest that instability of this model is more likely if technologies with increasing returns are introduced. As to that point, see also Brock (1988).

A final remark seems to be necessary concerning the formulation of the model in discrete time. As shown, we were able to demonstrate that erratic time paths of economic variables, frequently observed in reality, may result endogenously. But given the Li-Yorke theorem stating that period 3 implies chaos, and with due regard to the technical details, any *ad-hoc* postulated 'tent shape' difference equation may reveal erratic fluctuations for a certain parameter constellation. Although that theorem was proved only for one-dimensional systems, there is strong evidence that the coupled combination of two equations of that form may also lead to chaotic behaviour of the variables. For



that reason empirical research seems to be indispensable to gain insight into real-world economic phenomena showing aperiodic fluctuations in order to clarify whether they are merely the result of stochastic shocks or the outcome of nonlinear dynamics.

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